In general, that should be fine. In fact, I think that a) these estimates will be very, very close to the ones you would get using "svy", which you can confirm by re-running your original regressions that way; b) they should be more "conservative" estimates, in the sense that, by ignoring the stratification, they are likely to generate standard error and p-value estimates that are weakly too big... which is to say, if you get significance there, you are fine. The estimates will be unbiased for sure, because only the weighting needs to be accounted for to assure unbiasedness - getting appropriately sized SE/p-values requires clustering and stratification, but ignoring the stratification should make almost 0 difference (and like I said, you can check that by re-doing the estimates you've already done with "svy").

For the record -I promise I am not the referee that is making you do the extra analyses (smiley face), but if I were, I would be fine with this approach.